



**FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2022 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT**

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE:** (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C**. **ALL** questions carry **EQUAL** marks.
- (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v) Extra attempt of any question or any part of the attempted question will not be considered.
- (vi) **Use of Calculator is allowed.**

SECTION-A

- Q. 1. (a)** Let G be a group and H be a subgroup of index 2 in G . Show that H is normal in G . (10)
- (b)** Let G be any group, g a fixed element in G . Define $\phi: G \rightarrow G$ by $\phi(x) = gxg^{-1}, \forall x \in G$. Prove that ϕ is an automorphism of G onto G . (10) **(20)**
- Q. 2. (a)** Prove that a finite integral domain is a field. (10)
- (b)** Let W be the subspace of $\mathbb{R}^5$ spanned by $u_1 = (1, 2, -1, 3, 4), u_2 = (2, 4, -2, 6, 8), u_3 = (1, 3, 2, 2, 6), u_4 = (1, 4, 5, 1, 8), u_5 = (2, 7, 3, 3, 9)$. Find a subset of the vectors that form a basis of W . Also extend the basis of W to a basis of $\mathbb{R}^5$. (10) **(20)**
- Q. 3. (a)** Let $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be defined by $T(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$. Find the rank and nullity of T . (10)
- (b)** Find all possible solutions of the following homogeneous system of equations. (10) **(20)**
- $$\begin{aligned}x_1 + x_2 + x_3 - x_4 &= 0 \\x_1 + 2x_2 - 2x_3 + x_4 &= 0 \\2x_1 + 4x_2 - 3x_3 + x_4 &= 0 \\4x_1 + 7x_2 - 4x_3 + x_4 &= 0\end{aligned}$$

SECTION-B

- Q. 4. (a)** Find $\lim_{x \rightarrow \infty} (1 + 2x)^{1/(2 \ln x)}$. (10)
- (b)** Evaluate the integral $\int e^{3x} \cos 2x \, dx$. (10) **(20)**
- Q. 5. (a)** If $u = f(x, y)$ and $x = r \cos \theta, y = r \sin \theta$, then show that $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta}\right)^2$ (10)
- (b)** Evaluate $\iint_R x \, dx \, dy$ over the region bounded by $y = x^2$ and $y = x^3$. (10) **(20)**
- Q. 6. (a)** Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$. (10)
- (b)** Find the foci, vertices and center of the ellipse: $9x^2 + 16y^2 - 72x - 96y + 144 = 0$ (10) **(20)**

SECTION-C

Q. 7. (a) Prove that the function $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is harmonic. Also find a function $v(x, y)$ such that $f(z) = u(x, y) + i v(x, y)$ is analytic. (10)

(b) Evaluate $\oint_C \bar{z}^2 dz$ around the circle $|z| = 1$. (10) **(20)**

Q. 8. (a) Use residues to prove that (10)

$$\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$$

(b) Find the Fourier series of the following function $f(x)$ which is assumed to have the period 2π . (10) **(20)**

$$f(x) = |x|, -\pi < x < \pi$$
