



**FEDERAL PUBLIC SERVICE COMMISSION**  
**COMPETITIVE EXAMINATION-2021 FOR RECRUITMENT**  
**TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT**

Roll Number

**APPLIED MATHEMATICS**

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| <b>TIME ALLOWED: THREE HOURS</b>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | <b>MAXIMUM MARKS = 100</b> |
| <b>NOTE:</b> (i) Attempt <b>ONLY FIVE</b> questions. <b>ALL</b> questions carry <b>EQUAL</b> marks<br>(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.<br>(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.<br>(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.<br>(v) Extra attempt of any question or any part of the attempted question will not be considered.<br>(vi) <b>Use of Calculator is allowed.</b> |                            |

- Q. No. 1.** (a) Evaluate the surface integral  $\iint \vec{A} \cdot \vec{n} dS$  where  $\vec{A} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$  and  $S$  is the portion of the cylinder  $x^2 + y^2 = 8$  lying in the first octant between  $z = 0$  and  $z = 4$ . (10)

- (b) Prove that (10)
- $$\nabla(f(r)) = \frac{f'(r)}{r} \vec{r},$$
- where  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  and  $r = |\vec{r}|$ .

- Q. No. 2.** (a) The greatest resultant that two forces can have is of magnitude  $P$  and the least is of magnitude  $Q$ . Show that, when they act at an angle  $\alpha$ , their resultant is of magnitude  $\sqrt{P^2 \cos^2 \frac{\alpha}{2} + Q^2 \sin^2 \frac{\alpha}{2}}$ . (10)

- (b) A sphere of weight  $W$  and radius  $a$  is suspended by a string of length  $l$  from a point  $P$  and a weight  $w$  is also suspended from  $P$  by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is  $\sin^{-1} \frac{wa}{(W + w)(a + l)}$ . (10)

- Q. No. 3.** (a) Show that the law of force towards the pole, of a particle describing the curve  $r^n = a^n \cos n\theta$  is given by (10)

$$f = \frac{(n + 1)h^2 a^{2n}}{r^{2n+3}}.$$

- (b) The maximum velocity that a particle executing simple harmonic motion of amplitude  $a$  attains, is  $v$ . If it is disturbed in such a way that its maximum velocity becomes  $nv$ . Find the change in the amplitude and the time-period of motion. (10)

- Q. No. 4.** (a) Define ordinary and singular points of the differential equation  $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$ . When a singular point is said to be regular and irregular? Find regular and irregular singular points of the differential equation  $(x^2 - 4)^2 y'' + (x - 2)y' + y = 0$ . (10)

- (b) Show that (10)

$$J_{3/2} = \sqrt{\frac{2}{\pi x}} \left[ \frac{\sin x}{x} - \cos x \right].$$

- Q. No. 5.** (a) Solve the equation by using method of undetermined coefficients  $y'' - y' + y = 2 \cos 3x$ . (10)

- (b) Use the method of Frobenius to find two linear independent series solutions in powers of  $x$  of the DE.  $x^2 y'' - (x^2 + x)y' + y = 0$ . (10)

**Q. No. 6.** (a) Classify general second order partial differential equation (PDE) into elliptic, parabolic and hyperbolic form. Discuss the nature of the PDE  $(1 - x^2)u_{xx} - 2xyu_{xy} + (1 - y^2)u_{yy} = 0$  at each  $(x, y) \in R^2$ . (10)

(b) Use the method of separation of variables to find the solution  $u(x, t): [0, T] \times [0, L] \rightarrow R$  to the initial/boundary value problem  
 $u_t(x, t) = u_{xx}(x, t)$  for  $0 < t \leq T$  and  $0 \leq x \leq L$ ,  
 $u(x, 0) = f(x)$ , for  $0 \leq x \leq L$ ,  
 $u(0, t) = u(L, t) = 0$ , for  $0 < t \leq T$ ,  
where  $f: [0, L] \rightarrow R$  is a known function. (10)

**Q. No. 7.** (a) Use Simpson's 3/8 rule to estimate the integral  $\int_1^3 (x^3 - 2x^2 + 7x - 5)dx$ . (10)

By comparing your answer with exact value, find the error.

(b) Solve the system of equations by Jacobi iterative method. (10)  
 $10x + 3y + z = 19, \quad 3x + 10y + 2z = 29, \quad x + 2y + 10z = 35$

**Q. No. 8.** (a) In the following table values of  $y = x + \sin x^2$  are tabulated (10)

|        |         |         |         |         |         |         |         |
|--------|---------|---------|---------|---------|---------|---------|---------|
| $x$    | 1.0     | 1.1     | 1.2     | 1.3     | 1.4     | 1.5     | 1.6     |
| $f(x)$ | 1.84147 | 2.03562 | 2.19146 | 2.29290 | 2.32521 | 2.27807 | 2.14935 |

Construct a difference table and estimate  $f(1.04)$  and  $f(1.57)$ .

(b) Use trapezoidal and Simpson's 1/3 rules to approximate  $\int_0^{\pi/2} \sin^2(x)dx$ . Find a maximum bound for the error in each case. Compare your approximations with the actual result. (10)

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