



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2018
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT
PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE: (i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C**. **ALL** questions carry **EQUAL** marks.
- (ii)** All the parts (if any) of each Question must be attempted at one place instead of at different places.
- (iii)** Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
- (iv)** No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
- (v)** Extra attempt of any question or any part of the attempted question will not be considered.
- (vi)** **Use of Calculator is allowed.**

SECTION-A

- Q. 1. (a)** Let H and K be normal subgroups of a group G . Show that HK is a normal subgroup of G . (10)
- (b)** Let H and K be normal subgroups of a group G such that $H \subseteq K$. Then show that (10) (20)
- $$(G/H)/(K/H) \cong G/K$$
- Q. 2. (a)** Show that every finite integral domain is a field. (10)
- (b)** Consider the following linear system, (10) (20)
- $$\begin{aligned}x + 2y + z &= 3 \\ ay + 5z &= 10 \\ 2x + 7y + az &= b\end{aligned}$$
- (i)** Find the values of a for which the system has unique solution.
- (ii)** Find the values of the pair (a, b) for which the system has more than one solution.
- Q. 3. (a)** Find condition on a, b, c so that vector (a, b, c) in $\mathbb{R}^3$ belongs to (10)
- $$W = \text{span} \{u_1, u_2, u_3\} \text{ where } u_1 = (1, 2, 0), \quad u_2 = (-1, 1, 2), \quad u_3 = (3, 0, -4).$$
- (b)** Let W_1 and W_2 be finite dimensional subspaces of a vector space V . Show that (10) (20)
- $$\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$$

SECTION-B

- Q. 4. (a)** Let $f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ x & \text{if } x > 1 \end{cases}$ (10)
- Does the Mean Value Theorem hold for f on $\left[\frac{1}{2}, 2\right]$.
- (b)** Calculate the. $\lim_{x \rightarrow 0} \frac{\ln \sin 3x}{\ln \sin x}$ (10) (20)
- Q. 5. (a)** Evaluate $\int_{-1}^5 |x - 2| dx$. (10)
- (b)** Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$ if (10) (20)
- $$f(x, y) = \begin{cases} x^2 y \sin \frac{1}{x} & \text{when } x, y \text{ are not both } 0 \\ 0 & \text{when } x, y \text{ are both } 0 \end{cases}$$

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- Q. 6. (a)** Find the area of the region bounded by the cycloid $x = a(\theta - \sin \theta)$, $y = a(1 - \cos \theta)$ and its base. (10)
- (b)** Find the equation of a plane through (5,-1,4) and perpendicular to each of the planes $x + y - 2z - 3 = 0$ and $2x - 3y + z = 0$ (10) **(20)**

SECTION-C

- Q. 7. (a)** Express $\cos^5 \theta \sin^3 \theta$ in a series of sines of multiples of θ . (10)
- (b)** Use Cauchy's Residue Theorem to evaluate the integral $\int_C \frac{5z-2}{Z(Z-1)} dz$ where C (10) **(20)**
is the circle $|z| = 2$, described counter clock wise.
- Q. 8. (a)** Find the Laurent series that represent the function $f(z) = \frac{z+1}{z-1}$ in the domain $1 < |z| < \infty$. (10)
- (b)** Expand $f(x) = \sin x$ in a Fourier cosine series in the interval $0 \leq x \leq \pi$. (10) **(20)**
