FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2016
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Prove that the normaliser of a subset of a group G is a Subgroup of G. (10)

(b) Let A be a normal subgroup and B a subgroup of a group G. Then prove that $< A,B > = AB$ (10) **(20)**

Q. 2. **(a)** Let a be a fixed point of a group G and consider the mapping $I_a : G \to G$ defined by $I_a(g) = aga^{-1}$ where $g \in G$. (10)

Show that I_a is an automorphism of G. Also show that for $a, b \in G$, $I_a . I_b = I_{ab}$ (10) **(20)**

(b) Let $M_2(R) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R \right\}$ be the set of all 2×2 matrices with real entries. Show that $(M_2(R), +, \cdot)$ forms a ring with identity. Is $(M_2(R), +, \cdot)$ a field?

Q. 3. **(a)** Let $T: X \to Y$ be a linear transformation from a vector space X into a Vector space Y. Prove that Kernal of T is a subspace. (10)

(b) Find the value of λ such that the system of equations (10) **(20)**

$$x + \lambda y + 3z = 0$$
$$4x + 3y + \lambda z = 0$$
$$2x + y + 2z = 0$$

has non-trivial solution.

SECTION-B

Q. 4. **(a)** Using $\delta - \in$ definition of continuity, prove that the function $Sin^2 x$ is continuous for all $x \in R$. (10)

(b) Find the asymptotes of the curve $(x^2-y^2)(x+2y) + 5(x^2+y^2) + x+y = 0$ (10) **(20)**

Q. 5. **(a)** Prove that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$ (10)

PURE MATHEMATICS

Q. 6. **(a)** Find the area enclosed between the curves $y=x^3$ and $y=x$. (10)

(b) A plane passes through a fixed point (a, b, c) and cuts the coordinate axes in A, B, C. Find the locus of the centre of the sphere OABC for different positions of the plane, O is the origin. (10) **(20)**

SECTION-C

Q. 7. **(a)** Determine $P(z)$ where (10)

$P(z)=(z-z_1)(z-z_2)(z-z_3)(z-z_4)$ with $z_1=e^{i\pi/4}, z_2=\bar{z}_1, z_3=-z_1$ and $z_4=-\bar{z}_1$.

(b) Find value of the integral $\int_c (z-z_0)^n \, dz$, (n any integer) along the circle C ....with centre and z_0 radius r, described in the counter clock wise direction. (10) **(20)**

Q. 8. **(a)** Use Cauchy Integral Formula to evaluate $\int_c \frac{\cos h z + \sin 2z}{z - \pi/2} dz$ along the simple closed counter C: $|z|=3$ described in the positive direction. (10)

(b) State and prove Cauchy Residue Theorem. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let H, K be subgroups of a group G. Prove that HK is a subgroup of G if and only if $HK=KH$. (10)

(b) If N, M are normal subgroups of a group G, prove that (10) **(20)**

$$NM/M \cong N/N\cap M.$$

Q. 2. **(a)** If R is a commutative ring with unit element and M is an ideal of R then show that M is a maximal ideal of R if and only if R/M is a field. (10)

(b) If F is a finite field and $\alpha \neq 0, \beta \neq 0$ are two elements of F then show that we can find elements a and b in F such that (10) **(20)**

$$1 + \alpha a^2 + \beta b^2 = 0.$$

Q. 3. **(a)** Let V be a finite-dimensional vector space over a field F and W be a subspace of V. Then show that W is finite-dimensional, (10)

$$\dim W \leq \dim V \text{ and } \dim V/W = \dim V - \dim W.$$

(b) Suppose V is a finite-dimensional vector space over a field F. Prove that a linear transformation $T \in A(V)$ is invertible if and only if the constant term of the minimal polynomial for T is not 0. (10) **(20)**

SECTION-B

Q. 4. **(a)** Use the Mean-Value Theorem to show that if f is differentiable on an interval I, and if $|f'(x)| \leq M$ for all values of x in I, then (10)

$$|f(x) - f(y)| \leq M|x - y|$$

for all values of x and y in I. Use this result to show further that

$$|\sin x - \sin y| \leq |x - y|.$$

(b) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable at t, and if $z = f(x, y)$ is differentiable at the point $(x, y) = \left(x(t), y(t)\right)$, then $z = f\left(x(t), y(t)\right)$ is differentiable at t and (10) **(20)**

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y).

Q. 5. **(a)** Evaluate the double integral (10)

$$\iint_R (3x - 2y)\, dx\, dy$$

(b) Where R is a region enclosed by the circle $x^2 + y^2 = 1$. (10) **(20)**

Find the area of the region enclosed by the curves

$$y = \sin x, \quad y = \cos x, x = 0, \quad x = 2\pi.$$

Q. 6. **(a)** Find an equation of the ellipse traced by a point that moves so that the sum of its distance to (4,1) and (4,5) is 12. (10)

(b) Show that if a, b and c are nonzero, then the plane whose intercepts with the coordinate axes are $x = a$, $y = b$, and $z = c$ is given by the equation. (10) **(20)**

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$$

SECTION-C

Q. 7. **(a)** Prove that a necessary and sufficient condition that (10)

$$w = f(z) = u(x,y) + iv(x,y)$$

be analytic in a region R is that the Cauchy-Riemann equations

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}\text{and}\frac{\partial u}{\partial y}=-\frac{\partial v}{\partial x}$$

are satisfied in R where it is supposed that these partial derivatives are continuous in *R.*

(b) Show that the function $f(z) = \bar{z}$ is not analytic anywhere in the complex plane *Z.* (10) **(20)**

Q. 8. **(a)** Let $f(z)$ be analytic inside and on the boundary C of a simply-connected region R. Prove that (10)

$$f'(a)=\frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2}dz.$$

(b) Show that (10) **(20)**

$$\int_0^{2\pi}\frac{d\theta}{(5-3\sin\theta)^2}=\frac{5\pi}{32}.$$

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2018
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
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(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let H and K be normal subgroups of a group G. Show that HK is a normal subgroup of G. (10)

(b) Let H and K be normal subgroups of a group G such that $H \subseteq K$. Then show that (10) **(20)**

$$(G/H)\big/(K/H) \cong G/K$$

Q. 2. **(a)** Show that every finite integral domain is a field. (10)

(b) Consider the following linear system, (10) **(20)**

$$x + 2y + z = 3$$
$$ay + 5z = 10$$
$$2x + 7y + az = b$$

(i) Find the values of a for which the system has unique solution.

(ii) Find the values of the pair (a, b) for which the system has more than one solution.

Q. 3. **(a)** Find condition on a,b,c so that vector (a,b,c) in R^3 belongs to (10)

$W = span\{u_1,u_2,u_3\}$ where $u_1 = (1,2,0)$, $u_2 = (-1,1,2)$, $u_3 = (3,0,-4)$.

(b) Let W_1 and W_2 be finite dimensional subspaces of a vector space V. Show that (10) **(20)**

$$dimW_1 + dimW_2 = dim(W_1 \cap W_2) + dim(W_1 + W_2)$$

SECTION-B

Q. 4. **(a)** Let $f(x) = \begin{cases} x^2 & if \;\; x \le 1 \\ x & if \;\; x > 1 \end{cases}$ (10)

Does the Mean Value Theorem hold for f on $\left[\frac{1}{2}, 2\right]$.

(b) Calculate the. $\lim\limits_{x \to 0} \dfrac{ln\sin 3x}{ln\sin x}$ (10) **(20)**

Q. 5. **(a)** Evaluate $\int_{-1}^{5} |x-2|\,dx$. (10)

(b) Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$ if (10) **(20)**

$$f(x,y) = \begin{cases} x^2 y \sin\dfrac{1}{x} & when\ x, y\ are\ not\ both\ 0 \\ 0 & when\ x, y\ are\ both\ 0 \end{cases}$$

Q. 6. **(a)** Find the area of the region bounded by the cycloid (10)
$x = a(\theta - \sin\theta),\ \ y = a(1 - \cos\theta)$ and its base.

(b) Find the equation of a plane through (5,-1,4) and perpendicular to each of the planes (10) **(20)**

$$x + y - 2z - 3 = 0 \text{ and } 2x - 3y + z = 0$$

SECTION-C

Q. 7. **(a)** Express $\cos^5\theta\ \sin^3\theta$ in a series of sines of multiples of θ. (10)

(b) Use Cauchy's Residue Theorem to evaluate the integral $\int_C \frac{5z-2}{Z(Z-1)}dz$ where C is the circle $|z| = 2$, described counter clock wise. (10) **(20)**

Q. 8. **(a)** Find the Laurent series that represent the function $f(z) = \frac{z+1}{z-1}$ in the domain $1 < |z| < \infty$. (10)

(b) Expand $f(x) = sinx$ in a Fourier cosine series in the interval $0 \le x \le \pi$. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2019
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PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Show that the order and the index of a subgroup divides the order of a finite group. (10)

(b) Show that every finite integral domain is a field. (10) **(20)**

Q. 2. **(a)** Show that the characteristic of an integral domain is R is either zero or a prime. (10)

(b) Determine whether or not the set $\{(1,2,-1),(0,3,1),(1,-5,\ 3)\}$ of vectors is a basis for R^3. (10) **(20)**

Q. 3. **(a)** Show that a one-to-one linear transformation preserves basis and dimension. (10)

(b) Solve the system of linear equations: (10) **(20)**

$$2x_1 + x_2 + 5x_3 = 4$$
$$3x_1 - 2x_2 + 2x_3 = 2$$
$$5x_1 - 8x_2 + 2x_3 = 1.$$

SECTION-B

Q. 4. **(a)** Solve $\int_0^{\frac{\pi}{2}} \sin^2 6x \cos^4 3x\, dx.$ (10)

(b) Find the area enclosed by $y = \frac{6}{2-\cos\theta}$. (10) **(20)**

Q. 5. **(a)** Show that in any conic semi-latusrectum is the harmonic mean between the segments of focal chord. (10)

(b) Prove that the evolute of hyperbola (10) **(20)**

$$2xy = a \text{ is } (x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}.$$

Q. 6. **(a)** Define Supremum and Infimum of a sequence. Find the supremum and infimum of the set (10)

$$\left\{(-1)^n\left(1-\frac{1}{n}\right), n = 1,2,3\ldots\right\}.$$

(b) Evaluate (10) **(20)**

$$\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}}-e}{x}.$$

SECTION-C

Q. 7. **(a)** Show that $Log(1+\cos\theta+i\sin\theta)=\ln(2\cos\frac{\theta}{2})+i\frac{\theta}{2}$. (10)

(b) Find v such that $f(z)=u+iv$ is analytic. (10) **(20)**

Q. 8. **(a)** Prove that the series $z(1-z)+z^2(1-z)+z^3(1-z)+\cdots$ converges for $|z|<1$, and find its sum. (10)

(b) Find the residues of $f(z)=\frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2020
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let G and G' be two groups and $f: G \to G'$ be a homomorphism then prove the following: (10)

(i) $f(e) = e'$ where e and e' are the identities of G and G' respectively

(ii) $f(a^{-1}) = [f(\text{a})]^{-1}, \ \forall a \in G$

(b) Prove that every homomorphic image of a group is isomorphic to some quotient group. (10) **(20)**

Q. 2. **(a)** A ring R is without zero divisor if and only if the cancellation law hold. (10)

(b) Prove that arbitrary intersection of subrings is a subring. (10) **(20)**

Q. 3. **(a)** Let $\mathrm{T}: \mathrm{R}^3 \longrightarrow \mathrm{R}^3$ be the linear transformation defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 + x_3, x_2 + x_3)$. Find a basis and dimension of Range of T. (10)

(b) Prove that every finitely generated vector space has a basis. (10) **(20)**

SECTION-B

Q. 4. **(a)** Find the critical points of $f(x) = x^3 - 12x - 5$ and identify the open intervals on which f is increasing and on which f is decreasing. (10)

(b) Find the horizontal and vertical asymptotes of the graph of $f(x) = -\dfrac{8}{x^2 - 4}$ (10) **(20)**

Q. 5. **(a)** Calculate $\displaystyle\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$. (10)

(b) Find $\dfrac{\partial w}{\partial x}$ at the point $(x, y, z) = (2, -1, 1)$ if $w = x^2 + y^2 + z^2$, $z^3 - xy + yz + y^3 = 1$ and x and y are the independent variables. (10) **(20)**

Q. 6. **(a)** Determine the focus, vertex and directrix of the parabola $x^2 + 6x - 8y + 17 = 0$ (10)

(b) Find polar coordinates of the point p whose rectangular coordinates are $\left(3\sqrt{2}, -3\sqrt{2}\right)$ (10) **(20)**

SECTION-C

Q. 7. **(a)** Show that $\left(\cos\theta + i\sin\theta\right)^{n} = \cos(\mathrm{n}\theta) + \mathrm{i}\sin(\mathrm{n}\theta)$ for all integers *n*. (10)

(b) Find the *n*, *n*th roots of unity. (10) **(20)**

Q. 8. **(a)** Find the Taylor series generated by $f(\mathrm{x}) = \dfrac{1}{x}$ at $a = 2$. Where, if anywhere, does the series converge to $\dfrac{1}{x}$? (10)

(b) Show that the p-series $\sum_{n=1}^{\infty} \dfrac{1}{n^{p}}$, (*p* a real constant) converges if $p > 1$, and diverges if $P < 1$ (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2021
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

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(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let Ψ be a homomorphism of group G into group $\tilde{G}$ with kernel K, prove that K is a normal subgroup of G. (10)

(b) Prove that if H and K are two subgroups of a group G, then HK is a subgroup of G if and only if HK=KH. (10) **(20)**

Q. 2. **(a)** Find elements of the cyclic group generated by the permutation. (10)

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 2 & 6 & 1 \end{pmatrix}$$

(b) (10) **(20)**
Verify that the polynomials $2-x^2$, x^3-x, $2-3x^2$ and $3-x^3$ form a basis for the set $P_3(x)$; the set of all polynomials of degree three. Also express the vectors $1+x^2$ and $x+x^3$ as a linear combination of these basis vectors.

Q. 3. **(a)** Let V be the real vector space of all function from R to R. Show that $\{\cos^2 x, \sin^2 x, \cos 2x\}$ is linearly dependent while $\{\cos x, \sin x, \cosh x, \sinh x\}$ are linearly independent. (10)

(b) Solve the system of linear equations: (10) **(20)**

$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 + 11x_3 + 8x_4 &= 7 \end{aligned}$$

SECTION-B

Q. 4. **(a)** If $f(x,y) = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$. (10)

Show that $\frac{\partial^2 f}{\partial y \partial x}(x,y) = \left(\frac{x^2 - y^2}{x^2 + y^2}\right)$

(b) Evaluate $\int_0^6 f(x)dx$ where $f(x) = \begin{cases} x^2 \text{ when } x < 2 \\ 3x - 2 \text{ when } x > 2 \end{cases}$ (10) **(20)**

Q. 5. **(a)** Let $I_n = \int_0^\infty x^n e^{-x} dx$ where n is an integer. Prove that $I_n = n\, I_{n-1}$ Hence show that $I_n = n!$ (10)

(b) **i.** Write $r = \dfrac{8}{2-\cos\theta}$ in rectangular coordinates. (10) **(20)**

ii. Write $x^4 + 2x^2y^2 + y^4 - 6x^2y + 2y^3 = 0$ in polar coordinates.

Q. 6. **(a)** Evaluate $\iint_D dydx$ and $\iint_D dxdy$ where D is the region bounded by the y-axis, the lines x=2 and the curve e^x. (10)

(b) Investigate the curve $y = \dfrac{x^3 - x}{3x^2+1}$ for points of inflexion. (10) **(20)**

SECTION-C

Q. 7. **(a)** Sum the series $1 + \frac{1}{2}\cos\theta + \frac{1.3}{2.4}\cos 2\theta + \frac{1.3.5}{2.4.6}\cos 3\theta + \ldots$ (10)

(b) Prove that $\cos\frac{\pi}{7} - \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} = \frac{1}{2}$ (10) **(20)**

Q. 8. **(a)** Construct the analytic function f whose real part is $U = x^3 - 3xy^2 + 3x + 1$ (10)

(b) Evaluate $\int_C \dfrac{dz}{z^2 + 2z + 2}$ Where C is a square with corners (0,0),(-2,0),(-2,-2) and (0,-2). (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2022 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

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(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Let G be a group and H be a subgroup of index 2 in G. Show that H is normal in G. (10)

(b) Let G be any group, g a fixed element in G. Define $\phi: G \to G$ by $\phi(x) = gxg^{-1}, \forall x \in G$. Prove that ϕ is an automorphism of G onto G. (10) **(20)**

Q. 2. **(a)** Prove that a finite integral domain is a field. (10)

(b) Let W be the subspace of R^5 spanned by $u_1 = (1,2,-1,3,4), u_2 = (2,4,-2,6,8)$, $u_3 = (1,3,2,2,6), u_4 = (1,4,5,1,8), u_5 = (2,7,3,3,9)$. Find a subset of the vectors that form a basis of W. Also extend the basis of W to a basis of R^5. (10) **(20)**

Q. 3. **(a)** Let $T: R^4 \to R^3$ be defined by (10)

$$T(x,y,z,t) = (x - y + z + t,\ 2x - 2y + 3z + 4t,\ 3x - 3y + 4z + 5t)$$

Find the rank and nullity of T.

(b) Find all possible solutions of the following homogeneous system of equations. (10) **(20)**

$$x_1 + x_2 + x_3 - x_4 = 0$$
$$x_1 + 2x_2 - 2x_3 + x_4 = 0$$
$$2x_1 + 4x_2 - 3x_3 + x_4 = 0$$
$$4x_1 + 7x_2 - 4x_3 + x_4 = 0$$

SECTION-B

Q. 4. **(a)** Find $\lim_{x\to\infty} (1 + 2x)^{1/(2\ln x)}$. (10)

(b) Evaluate the integral $\int e^{3x} \cos 2x \, dx$. (10) **(20)**

Q. 5. **(a)** If $u = f(x,y)$ and $x = r\cos\theta, y = r\sin\theta$, then show that (10)

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

(b) Evaluate $\iint_R x \, dx \, dy$ over the region bounded by $y = x^2$ and $y = x^3$. (10) **(20)**

Q. 6. **(a)** Find the area of the region bounded above by $y = x + 6$, bounded below by $y = x^2$, and bounded on the sides by the lines $x = 0$ and $x = 2$. (10)

(b) Find the foci, vertices and center of the ellipse: (10) **(20)**

$$9x^2 + 16y^2 - 72x - 96y + 144 = 0$$

SECTION-C

Q. 7. **(a)** Prove that the function $u(x, y) = e^{-x}(x \sin y - y \cos y)$ is harmonic. Also find a function $v(x, y)$ such that $f(z) = u(x, y) + i\, v(x, y)$ is analytic. (10)

(b) Evaluate $\oint_C \bar{z}^2 dz$ around the circle $|z| = 1$. (10) **(20)**

Q. 8. **(a)** Use residues to prove that (10)

$$\int_0^{\infty} \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$$

(b) Find the Fourier series of the following function $f(x)$ which is assumed to have the period 2π. $f(x) = |x|, -\pi < x < \pi$ (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Find centre of S_3. (10)

(b) Using the row operations, show that the matrix $\begin{pmatrix} 1 & 2 & -3 \\ 1 & -2 & 1 \\ 5 & -2 & -3 \end{pmatrix}$ has no inverse. (10) **(20)**

Q. 2. **(a)** For any group G, show that $\frac{G}{\{e\}} \cong G$ and $\frac{G}{G} \cong \{e\}$. (10)

(b) Suppose U and W are distinct four dimensional subspaces of a vector space V of dimension six. Find the possible dimension of $U \cap W$. (10) **(20)**

Q. 3. **(a)** For what value of α is the matrix $\begin{pmatrix} -\alpha & \alpha-1 & \alpha+1 \\ 1 & 2 & 3 \\ 2-\alpha & \alpha+3 & \alpha+7 \end{pmatrix}$ is singular? (10)

(b) Define $T:\Re^3 \to \Re^3$ by $T(x_1, x_2, x_3) = (-x_3, x_1, x_1 + x_3)$. Find $N(T)$. Is T one-to-one? (10) **(20)**

SECTION-B

Q. 4. **(a)** Find the value of θ and the limit in order that $\lim\limits_{x\to 0}\frac{\sin 2x + \theta \sin x}{x^3}$ be finite. (10)

(b) Show that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$, 0<x<1. (10) **(20)**

Q. 5. **(a)** Given that $U = \frac{1}{\sqrt{x^2+y^2+z^2}}$. Verify that $U_{xx} + U_{yy} + U_{zz} = 0$. (10)

(b) Evaluate $\iint (x^2 + y^2)dxdy$, over the domain bounded by $y = x^2$ and $x = y^2$. (10) **(20)**

Q. 6. **(a)** Evaluate $\iint (x^2 + y^2)dxdy$, over the region bounded by xy=1, y=0, y=x and x=2. (10)

(b) Find an equation of a normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in the form $ax\cos\theta + by\cot\theta = a^2 + b^2$. Prove that the normal is external bisector of the angle between the focal distances of its foot. (10) **(20)**

SECTION-C

Q. 7. **(a)** Determine k such that $U = e^{2x}\cos ky$ is harmonic and find a conjugate harmonic. (10)

(b) Evaluate $\int_C (\frac{1}{z^5} + z^3)dz$ from 1 to -1 along the upper arc of the unit circle. (10) **(20)**

Q. 8. **(a)** Find the Laurent Series of $\frac{1}{1-z^2}$ in the region $0<|z-1|<2$. (10)

(b) Find the residues at the singular points of $\frac{-Z^2-22z+8}{Z^3-5z^2+4z}$ which lie inside the circle $|z|=2$. (10) **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION-2024 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. No.1.(a) Let N be a normal subgroup of a group G. If H is a subgroup of G, then prove that **(10)**

$NH = \{nh : n \in N and h \in H\}$ is a subgroup of G.

(b) If $\boldsymbol{E}$ is an epimorphism from a group G onto a group H then prove that G/K is isomorphic to H, where $K = Ker\boldsymbol{E}$. **(10) (20)**

Q. No.2. (a) Let R be a ring. If every $x \in R$ satisfies $x^2 = x$ then prove that R is a commutative. **(10)**

(b) For which value(s) of a will the following system have no solution? Exactly one solution? Infinitely many solutions? **(10) (20)**

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + \left(a^2 - 14\right) z = a + 2.$$

Q. No.3. (a) Determine a basis for and the dimension of the solution space of the system **(10)**

$$x - 2z + w = 0$$
$$3x + y - 5z = 0$$
$$x + 2y - 5w = 0.$$

(b) Let $v_1 = (1, 1, 1)$, $v_2 = (1, 1, 0)$ and $v_3 = (1, 0, 0)$ be a basis for $\mathbb{R}^3$. Find a linear transformation $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ such that $T(v_1) = (1, 0)$, $T(v_2) = (2,-1)$ and $T(v_3) = (4, 3)$. **(10) (20)**

SECTION-B

Q. No.4. (a) Evaluate the limit: **(10)**

(i) $\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{\tan x}$

(ii) $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

(b) State and prove the Mean Value Theorem. **(10) (20)**

Q. No.5. (a) If $w = f(x^2 + y^2)$ then show that $y\left(\frac{\partial w}{\partial x}\right) - x\left(\frac{\partial w}{\partial y}\right) = 0\cdot$ **(10)**

(b) Find all the local maxima, local minima and saddle points of the given function $2x^3 + y^2 - 9x^2 - 4y + 12x - 2\cdot$ **(10) (20)**

PURE MATHEMATICS

Q. No.6. (a) Evaluate the integral $\int_0^\infty x^{\frac{3}{2}}((1+2x))^{-5}\,dx$ and show that the result is $\frac{9\pi}{384}$, using **Beta** function. **(10)**

(b) Find the vertices and foci of the hyperbola **(10)** **(20)**

$$25x^2-16y^2+250x+32y+109=0.$$

SECTION-C

Q. No.7. (a) Verify that $u(x,y)=cosxcoshy$ is harmonic function and find a corresponding analytic function $f(z)=u(x,y)+iv(x,y)$. **(10)**

(b) Use Residue theorem to evaluate the integral $\int_C \frac{3z^2+z-1}{(z^2-1)(z-3)}\,dz$, where C is the circle $|z|=4$. **(10)** **(20)**

Q. No.8. (a) Use the Cauchy's integral formula to evaluate the integral (10) **(10)**

$$\int_C \frac{z+4}{z^2+2z+5}\,dz, \text{ where } C \text{ is the circle } |z+1-i|=2.$$

(b) Find the three cube roots of $\sqrt{3}+i$. **(10)** **(20)**

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FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2025 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT
PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: **(i)** Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. No.1.(a) Let Q^+ be the set of positive rational numbers and define $*$ by $a * b = \frac{ab}{2}$ then prove that $(Q^+, *)$ is a group. **(10)**

(b) Find all the cyclic subgroups of Z_{18}. **(10)**

Q. No.2. (a) A homomorphism $\varphi: Z_6 \to Z_6$ is a one to one mapping then calculate Ker(φ). **(10)**

(b) Check whether the vectors $a = (1, 2, 3), b = (2, 5, 7) and\ c = (1, 3, 5)$ are linearly dependent or independent. **(10)**

Q. No.3. (a) Let V be a vector space of all 2× 2 matrices. W be a subspace of V which consists of all symmetric matrices then find two different basis of W. **(10)**

(b) Consider the transformation $T: R^3 \to R^2$ given by $T(x, y, z) = (|x|, y+z)$. Check whether T is linear or not. **(10)**

SECTION-B

Q. No.4. (a) Find all the real numbers $x \in R$ such that $x^2 + x > 2$. **(10)**

(b) Use the definition of limit to establish that $\lim_{x \to 2} \frac{x^3 - 4}{x^2 + 1} = \frac{4}{5}$. **(10)**

Q. No.5. (a) Use Mean Value Theorem to show that $e^x \geq 1 + x \ \forall \ x \varepsilon R$. **(10)**

(b) Find the absolute extrema of the function $f(x, y) = xy - 2x$ on the region R given by vertices (0, 4), (4, 0) and (0, 0). **(10)**

Q. No.6. (a) Change the order of integration in double integral $\int_0^2 \int_0^{\sqrt{x}} f(x, y) dydx$. **(10)**

(b) Draw the graph of the conic $r = 2Cos\theta$. **(10)**

SECTION-C

Q. No.7. (a) Check that the Cauchy-Riemann Equations are satisfied in polar coordinates for $f(Z) = \frac{1}{Z}$. **(10)**

(b) Evaluate the contour integral $\oint_C f(Z)dZ$ where $f(Z) = y - x - 3ix^2$ and C is simple closed contour OABO with O = 0+0i, A = 0+i and B = 1+i. **(10)**

Q. No.8. (a) Use Cauchy Residue Theorem to evaluate the integral $\int_C \frac{5Z-2}{Z(Z-1)} dZ$ where C is the circle $|Z| = 2$. **(10)**

(b) Find the Maclaurin series for the function $f(Z) = Z^2 e^{3Z}$. **(10)**

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FEDERAL PUBLIC SERVICE COMMISSION

COMPETITIVE EXAMINATION-2026 FOR RECRUITMENT TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt FIVE questions in all by selecting TWO Questions each from SECTION-A&B and ONE Question from SECTION-C. ALL questions carry EQUAL marks.

(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) Use of Calculator is allowed.

SECTION-A

Q. No. 1. **(a)** State the Lagrange's Theorem and find all the subgroups of the cyclic group of order 24. Show that any group of prime order has no non-trivial subgroups. **(10)**

(b) Let G be an Abelian group and H is a subset of G consisting of those elements of G which are of the second order then H is a subgroup of G. Let $G = \{\begin{vmatrix} a & b \\ c & d \end{vmatrix} : a, b, c, d \in \mathbb{R}, ad - bc \neq 0\}$ be the group of all matrices under matrix multiplication. Show that $H = \{\begin{vmatrix} a & b \\ 0 & d \end{vmatrix} : a, b, d \in \mathbb{R}, ad \neq 0\}$ is a subgroup of G and $K = \{\begin{vmatrix} 1 & b \\ 0 & 1 \end{vmatrix} : b \in \mathbb{R}\}$ is a subgroup of H. **(10)**

Q. No. 2. **(a)** Consider the group $\mathbb{Z}_6$ under addition modulo 6. Let $\psi(x) = 5x \pmod 6$. Verify that it is an automorphism. Define Kernel and Image with at least one example of each. Consider the homomorphism: $\psi : \mathbb{Z}_4 \to \mathbb{Z}_2$, where $\psi(x) = x \pmod 2$. Find $Ker(\psi)$? Let $\text{Ø} : G \to H$ be a group homomorphism, and let $g \in G$. Then for any integer n, we have $\text{Ø}(g^n) = (\text{Ø}(g))^n$. **(10)**

(b) A computer chip manufacturer decides to spend 6×10^5 rupees on radio, magazine, and TV advertisement. If he spends as much on TV advertisement as on magazine and radio together and the amount spent on magazine and TV combined equals five times that spent on radio. What is the amount to be spent on each type of advertisement? Use Gauss Jordan Method. **(10)**

Q. No. 3. **(a)** Find Echelon form and Rank of the following matrix by successively reducing it to its row/column sub-matrices: **(10)**

$$\begin{vmatrix} 1 & -2 & 1 & 0 & 5 \\ -1 & 0 & 1 & -2 & 2 \\ 2 & -3 & 0 & 2 & 3 \end{vmatrix}$$

(b) Define linearly independent and linearly dependent vectors. Are the vectors $(1, -2, 4, 1)$, $(2, 1, 0, -3)$ and $(1, -6, 1, 4)$ in $\mathbb{R}^4$ linearly independent or linearly dependent? **(10)**

SECTION-B

Q. No. 4. **(a)** Consider two planes as follows: **(10)**

$P_1 : 2x - y - 2z + 5 = 0,$

$P_2 : 4x - 2y - 4z + 15 = 0.$

Are the planes parallel? If no, what is angle between them?

(b) Derive the equation of sphere in standard form. Find the equation of sphere through the circle: **(10)**

$$x^2 + y^2 + z^2 = 9, \quad 2x + 4y + 5z = 6$$

and touching the xz-plane.

PURE MATHEMATICS (2026)

Q. No. 5. **(a)** Find cylindrical coordinates of the point with rectangular coordinates $(2\sqrt{3}, 2, -2)$. Express the equation $(x + y)^2 - z^2 + 4 = 0$ in cylindrical and spherical coordinates. **(10)**

(b) Use Maclaurin Series to prove that: **(10)**

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots, \quad \textit{hence find the value of } \pi.$$

Q. No. 6. **(a)** Evaluate the following limits: **(10)**

(i) $\lim_{x \to 0} \left[\frac{1}{\sin 3x} - \frac{1}{3x} \right]$ (ii) $\lim_{x \to 0} \frac{x - \sin x}{x^3}$

(b) What is the area A, of the region bounded by the curves $y = x^2$ and $y = \sqrt{x}$? **(10)**

SECTION-C

Q. No. 7. **(a)** State De-Moivre's Theorem. If $x + \frac{1}{x} = 2\cos\theta$, $y + \frac{1}{y} = 2\cos\varphi$, $z + \frac{1}{z} = 2\cos\psi$, prove that $xyz + (xyz)^{-1} = 2\cos(\theta + \varphi + \psi)$. **(10)**

(b) State Green's Theorem and verify it for $\oint_C (2xy - x^2)\,dx + (x + y^2)\,dy$, where C is the curve bounded by $y = x^2$ and $x = y^2$. **(10)**

Q. No. 8. **(a)** Use the concept of Path Integrals to prove following: **(10)**

(i) $\int_{|z|=1} \frac{1}{z}\,dz = 2\pi i$ (ii) $\int_{|z|=1} z\,dz = 0$

(b) State Residue Theorem and use it to prove the following: **(10)**

$$\oint_C \frac{e^{-z}}{(z + 2)^5}\,dz = \frac{1}{12}\pi i\, e^2 \quad \textit{where C is the circle } |z| = 3.$$

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