



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2016
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100
NOTE:(i) Attempt ONLY FIVE questions. ALL questions carry EQUAL marks (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places. (iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper. (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed. (vi) Extra attempt of any question or any part of the attempted question will not be considered. (v) Use of Calculator is allowed.	

Q. No. 1. (a) Prove that $\nabla \cdot \left[\frac{f(r)\vec{r}}{r} \right] = \frac{2}{r} f(r) + f'(r)$ (10)

(b) Verify Stokes' theorem for $\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. (10)

Q. No. 2. (a) Forces P, Q, R act at a point parallel to the sides of a triangle ABC taken in the same order. Show that the magnitude of the resultant force is (10)

$$\sqrt{P^2 + Q^2 + R^2 - 2QR \cos A - 2RP \cos B - 2PQ \cos C}$$

(b) Find the distance from the cusp of the centroid of the region bounded by the cardioid $r = a(1 + \cos \theta)$. (10)

Q. No. 3. (a) A particle describes simple harmonic motion in such a way that its velocity and acceleration at a point P are u and f respectively and the corresponding quantities at another point Q are v and g. Find the distance PQ. (10)

(b) Derive the radial and transverse components of velocity and acceleration of a particle. (10)

Q. No. 4. Solve the following differential equations:

(a) $\frac{dy}{dx} + \frac{y}{x} = x^3 y^4$ (10)

(b) $(D^2 - 5D + 6)y = x^3 e^{2x}$ (10)

Q. No. 5. (a) Solve the differential equation using the method of variation of parameters (10)

$$\frac{d^2 y}{dx^2} + y = \tan x, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(b) Solve the Euler – Cauchy differential equation $x^2 y'' - 3xy' + 4y = x^2 \ln x$. (10)

Q. No. 6. (a) Find the Fourier series of the following function: (10)

$$f(x) = \begin{cases} -x & \text{if } -\pi < x < 0 \\ x & \text{if } 0 < x < \pi \end{cases}$$

(b) Solve the initial - boundary value problem: (10)

- Q. No. 7.** (a) Apply Newton – Raphson method to find the smaller positive root of the equation $x^2 - 4x + 2 = 0$ (10)
- (b) Solve the following system of equations by Gauss – Seidel iterative method by taking the initial approximation as $x_1 = 0, x_2 = 0, x_3 = 0$: (10)
- $$\begin{aligned} 5x_1 + x_2 - x_3 &= 4 \\ x_1 + 4x_2 + 2x_3 &= 15 \\ x_1 - 2x_2 + 5x_3 &= 12 \end{aligned}$$

- Q. No. 8.** (a) Approximate $\int_0^1 \frac{dx}{1+x^2}$ using (10)
- (i) Trapezoidal rule with $n = 4$ (ii) Simpson's rule with $n = 4$
Also compare the results with the exact value of the integral.
- (b) Apply the improved Euler method to solve the initial – value problem: (10)
- $$y' = x + y, \quad y(0) = 0$$
- by choosing $h = 0.2$ and computing $y_1, \dots, y_5$.



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Q. No. 1. (a) Suppose $\mathbf{r}(t) = (e^t \cos t)\hat{i} + (e^t \sin t)\hat{j}$. Show that the angle between $\mathbf{r}$ and $\frac{d^2\mathbf{r}}{dt^2}$ (10)
never changes. What is the angle?

(b) Using divergence theorem of Gauss, Evaluate $\iint_S x^2 dx dy + x^2 y dz dx + x^2 z dx dy$ (10)
where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$, ($0 \leq z \leq b$)
and the circular disks $z = 0$ and $z = b$, ($x^2 + y^2 \leq a^2$).

Q. No. 2. (a) A 100 Kg wooden crate rests on a wooden ramp with an adjustable angle of inclination. Draw a free body diagram of the crate. If the angle of the ramp is set to 10° , find the perpendicular and parallel components of the crate's weight to the ramp. Also find the static friction force between the crate and the ramp. At what angle will the crate just begin to slip? (coefficient of static friction μ_s between wood block and wood surface is 0.28) (10)

(b) A ladder having a uniform density and a mass m rests against a frictionless vertical wall at an angle of 60° . The lower end rests on a flat surface where the coefficient of static friction is 0.40. A person of mass $M=2m$ attempts to climb the ladder. What fraction of the length L of the ladder will the person have reached when the ladder begins to slip? (10)

Q. No. 3. (a) A particle moving in a straight line starts with a velocity u and has acceleration v^3 , where v is the velocity of the particle at time t . Find the velocity and the time as functions of the distance travelled by the particle. (10)

(b) A particle describing simple harmonic motion has velocities 5 ft/sec and 4 ft/sec (10)
when its distance from the centre are 12 ft and 13 ft respectively. Find the time-period of motion.

Q. No. 4. (a) Check for the exactness and solve the following ordinary differential equation. (10)
 $(2xy + y - \tan y)dx + (x^2 - x \tan^2 y + \sec^2 y)dy = 0$

(b) Solve the following second order differential equation: (10)
 $\frac{d^2 y}{dx^2} + 4x = \sec 2x$

Q. No. 5. (a) Solve the initial value problem. (10)
 $\frac{d^2 y}{dx^2} = 2-6x$, $y'(0) = 4$, $y(0) = 1$

(b) Solve the following Boundary Value Problem. (10)
 $y' + 4y = 0$, $y(0) = -2$, $y(2\pi) = -2$
How many solutions do you get for this problem?

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Q. No. 6. (a) Compute the Fourier series for the function x^2 on the interval $0 < x < L$, using as a basis of function with boundary conditions $u'(0) = 0$ and $u'(L) = 0$. Sketch the partial sums of the series for 1, 2, 3 terms. **(10)**

(b) Find a solution to the following partial differential equation that will also satisfy the boundary conditions. **(10)**

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad u(x,0) = f(x), \quad u(0,t) = 0, \quad u(L,t) = 0$$

Q. No. 7. (a) Use bisection and false position methods to locate the root of $f(x) = x^{10} - 1$, between 0 and 1.3. Which of the two methods is better and why? **(10)**

(b) Use Lagrange interpolation polynomial of the first and second order to evaluate $f(x)$ at $x=15$ for the following data; **(10)**

x:	0	20	40
f(x):	3.850	0.800	0.212

Q. No. 8. (a) Use Trapezoidal Rule with 3 segments to evaluate. **(10)**

$$\int_0^{0.8} x \cdot e^{2x} dx$$

Also find the true solution and the percentage error.

(b) Consider the function $f(x) = xe^x$. Obtain approximations to $f'(2)$ with $h = 0.5$ using forward, backward and central difference formulas. **(10)**



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APPLIED MATHEMATICS

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Q. No. 1. (a) If $\psi = \sin \frac{(kr)}{r}$, then show that $\nabla^2 \psi + k^2 \psi = 0$. **(10)**

(b) Calculate the Line Integral $\int_C A dr$, where $A = \frac{-yi + xj}{x^2 + y^2}$, and the curve C is **(10)**

given by the equations $x^2 + y^2 = a^2$ and $Z = 0$.

Q. No. 2. (a) Forces of magnitude P, 2P, 3P, 4P act respectively along the sides AB, BC, CD, DA of a square ABCD, of sides **a**, and forces each of magnitude $(8\sqrt{2})$ P act along the diagonals BD, AC. Find the magnitude of the resultant force and distance of its line of action from A. **(10)**

(b) A uniform ladder, of length 70 feet, rests against a vertical wall with which it makes an angle of 45° , the coefficient of friction between the ladder and the wall and the ground respectively being $\frac{1}{3}$ and $\frac{1}{2}$. If a man, whose weight is one half that of the ladder, ascends the ladder, where will he be when the ladder slips? **(10)**

Q. No. 3. (a) A particle moves in a straight line with an acceleration kv^3 . If its initial velocity is **u**, find the velocity and the time spent when the particle has travelled a distance **x**. **(10)**

(b) Derive the Tangential and Normal components of the velocity and acceleration. **(10)**

Q. No. 4. (a) Solve the following Cauchy- Euler Equation **(10)**

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = 0.$$

(b) Convert the following Bernoulli Differential Equation into standard form and then solve. **(10)**

$$\frac{dy}{dx} + \left(\frac{xy}{1 - x^2} \right) = xy^{\frac{1}{2}}.$$

Q. No. 5. (a) Convert the following Ordinary Differential Equation into standard form and then solve using Method of Variation of Parameters. **(10)**

$$x^2 y'' - 3xy' + 3y = 2x^4 e^x$$

(b) Check whether the following Ordinary Differential Equation is an Exact Equation or not. If yes, then solve. **(10)**

$$(3x^2 y + 2) dx + (x^3 + y) dy = 0$$

- Q. No. 6. (a)** Find the Fourier Series of f on the given interval. (10)

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 \leq x < \pi \end{cases}$$

- (b)** Solve the following Partial Differential Equation subject to the conditions given. (10)

$$a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \quad 0 < x < L, \quad t > 0,$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad t > 0,$$

$$u(x, 0) = f(x), \quad \frac{\partial u}{\partial t} = g(x) \text{ [at time } t = 0], \quad \text{and } 0 < x < L.$$

- Q. No. 7. (a)** Use Newton-Raphson method to find solution accurate to within 10^{-4} for the non-linear equation. (10)

$$x^3 - 2x^2 - 5 = 0, \quad I = [1, 4]$$

- (b)** Use Lagrange Interpolating polynomial of degree two to approximate $f(8.4)$, If (10)

$$f(8.1) = 16.94410, \quad f(8.3) = 17.56492, \quad f(8.6) = 18.50515, \quad f(8.7) = 18.82091.$$

- Q. No. 8. (a)** Approximate $\int_0^1 \frac{dx}{1+x^2}$ using Trapezoidal rule and Simpson's rule with $n=4$. (10)

Also compare your results with the exact value of the integral.

- (b)** Use Euler's method to approximate the solution of the following initial value problem. (10)

$$y' = \frac{1+y}{t}, \quad 1 \leq t \leq 2, \quad \text{with } y(1) = 2, \quad h = 0.25$$



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APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

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(vi) **Use of Calculator is allowed.**

Q. No. 1. (a) Find the directional derivative of $f(x, y, z) = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $i + 2j + 2k$? **(10)**

(b) Evaluate $\int_c (xy + y^2)dx + x^2dy$ where c is bounded by the line $y = x$ and the curve $y = x^2$ **(10)**

Q. No. 2. (a) Find the constants a , b , and c so that **(10)**
 $F = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$
is irrotational and hence find the function ψ such that $F = \nabla \psi$

(b) The forces F_1, F_2, F_3, F_4, F_5 and F_6 act along the sides of a regular hexagone taken in order. Verify that all the forces will be in equilibrium if, **(10)**
 $\sum F = 0$, and $F_1 - F_4 = F_3 - F_6 = F_5 - F_2$.

Q. No. 3. (a) A system of forces acts on a plate in the form of an equilateral triangle of side $2a$. The moment of the forces about the three vertices are M_1, M_2 and M_3 respectively. Find the magnitudes of the resultant. **(10)**

(b) If a particle P move with a velocity V given by $V^2 = n^2(ax^2 + 2bx + c)$. Show that P executes a simple harmonic motion. Find the centre, the amplitude and the time period of the motion? **(10)**

Q. No. 4. (a) What is the difference between linear differential equation and Bernoulli's equation? Also find the solution of the following differential equation. **(10)**

$$x \left[\frac{dy}{dx} + y \right] = 1 - y$$

(b) Use the method of undetermined coefficient to solve the following differential equation. **(10)**

$$y'' - 3y' + 2y = 2x^3 - 9x^2 + 6x$$

Q. No. 5. (a) Solve the equation **(10)**

$$0 = \frac{1}{2} + \frac{1}{4}x^2 - x \sin x - \frac{1}{2} \cos 2x \quad \text{with } x_0 = \frac{\pi}{2}$$

(b) Derive two point Gaussian integration formula for the following integral and use it to solve the integral. **(10)**

$$\int_1^{1.6} \frac{2x}{x^2 - 4} dx$$

- Q. No. 6. (a) Determine the second degree polynomials by using Newton's method. Also estimate the value of $f(0.1)$ and $f(0.5)$ for the data. (10)

x	0.0	0.2	0.4	0.6
$f(x)$	15.0	21.0	30.0	51.0

- (b) Does the dominate diagonal is necessary for finding the numerical solution of system of linear equations by using Gauss Jacobi's and Gauss Seidal methods. Explain the reason. In what conditions a numerical method is used instead of analytical method? Find the solution of the following system by performing three itrations of Gauss Seidal method. (10)

$$\begin{aligned} 6x - 3y + z &= 11 \\ 2x + y - 8z &= 15 \\ x - 7y + z &= 10 \end{aligned}$$

- Q. No. 7. (a) Define even function and odd function with examples. Verify that the Fourier (10)

Series for the function $f(x) = \begin{cases} 0 & \text{When } 0 < x < \pi \\ 1 & \text{When } \pi < x < 2\pi \end{cases}$

is $f(x) = \frac{1}{2} - \frac{2}{\pi}(\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x.....)$

- (b) Solve the following partial differential equation by using method of separable variable. (10)

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \quad \text{given} \quad u(x, 0) = 6e^{-3x}$$

- Q. No. 8. (a) The Trapezoidal rule applied to $\int_0^2 f(x)dx$ gives the value 4, and the Simpson's rule gives value 2, what is the value of $f(1)$? (10)

- (b) Find the first two derivatives at $x=1.1$ and $x =1$ from the following data table. (10)

x	1	1.2	1.4	1.6	1.8	2.0
$f(x)$	0.000	0.1280	0.5440	1.2960	2.4320	4.000



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APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks

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(v) Extra attempt of any question or any part of the attempted question will not be considered.

(vi) **Use of Calculator is allowed.**

Q. No. 1. (a) Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ **(10)**

(b) Evaluate $\int_S \underline{A} \cdot \underline{n} \, ds$ where $\underline{A} = 18z\underline{i} - 12\underline{j} + 3y\underline{k}$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the 1st octant. **(10)**

Q. No. 2. A particle P of mass m slides down a frictionless inclined plane AB of an angle α with the horizontal. If it starts from rest at the top A, find (a) the acceleration (b) the velocity and (c) the distance travelled after time t. **(20)**

Q. No. 3. (a) Discuss the motion of a particle moving in a straight line if it starts from rest at a distance 'a' from a point O and moves with an acceleration equal to k times its distance from O. **(10)**

(b) Find radial and transversal components of velocity and acceleration. **(10)**

Q. No. 4. (a) Solve $\frac{d^2 y}{dx^2} + y = \text{Cosec } x$ **(10)**

(b) Solve $dy + \frac{y - \sin x}{x} dx = 0$ **(10)**

Q. No. 5. (a) Solve the initial value problem **(10)**
 $x(2+x)\frac{dy}{dx} + 2(1+x)y = 1 + 3x^2, \quad y(-1) = 1$

(b) Find the general solution of the equation **(10)**
 $(D^3 - 2D + 1)y = 2x^3 - 3x^2 + 4x + 5$

Q. No. 6. (a) Find the Fourier series of f: **(10)**
$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$

(b) Solve the boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}$ **(10)**
Satisfying $u(0,t) = u(l,t) = 0$ and $u(x,0) = lx - x^2$

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- Q. No. 7.** (a) By using regular Falsi method, solve **(10)**
 $\text{Log}x - \text{Cos}x = 0$
- (b) Find the value of $f(7.5)$ by using Newton Gregory Backward Difference **(10)**
Interpolation formula.
X: 5, 6.1, 6.9, 8, 8.6
 $f(x): 3.49, 4.82, 5.96, 7.5, 8.2$
- Q. No. 8.** (a) Applying the Taylor series method, compute **(10)**
 $\int_0^x \frac{\sin t}{t} dt$ for $x = 0$ (0.1)1
- (b) Use fourth order RK method to solve **(10)**
 $\frac{dy}{dx} = t + y$; $y(0) = 1$ from $t = 0$ to $t = 0.4$ taking $h = 0.4$



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- Q. No. 1.** (a) Evaluate the surface integral $\iint \vec{A} \cdot \vec{n} dS$ where $\vec{A} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and S is the portion of the cylinder $x^2 + y^2 = 8$ lying in the first octant between $z = 0$ and $z = 4$. (10)

- (b) Prove that (10)
- $$\nabla(f(r)) = \frac{f'(r)}{r} \vec{r},$$
- where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$.

- Q. No. 2.** (a) The greatest resultant that two forces can have is of magnitude P and the least is of magnitude Q . Show that, when they act at an angle α , their resultant is of magnitude $\sqrt{P^2 \cos^2 \frac{\alpha}{2} + Q^2 \sin^2 \frac{\alpha}{2}}$. (10)

- (b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is $\sin^{-1} \frac{wa}{(W+w)(a+l)}$. (10)

- Q. No. 3.** (a) Show that the law of force towards the pole, of a particle describing the curve $r^n = a^n \cos n\theta$ is given by (10)

$$f = \frac{(n+1)h^2 a^{2n}}{r^{2n+3}}.$$

- (b) The maximum velocity that a particle executing simple harmonic motion of amplitude a attains, is v . If it is disturbed in such a way that its maximum velocity becomes nv . Find the change in the amplitude and the time-period of motion. (10)

- Q. No. 4.** (a) Define ordinary and singular points of the differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$. When a singular point is said to be regular and irregular? Find regular and irregular singular points of the differential equation $(x^2 - 4)^2 y'' + (x - 2)y' + y = 0$. (10)

- (b) Show that (10)

$$J_{3/2} = \sqrt{\frac{2}{\pi x}} \left[\frac{\sin x}{x} - \cos x \right].$$

- Q. No. 5.** (a) Solve the equation by using method of undetermined coefficients $y'' - y' + y = 2 \cos 3x$. (10)

- (b) Use the method of Frobenius to find two linear independent series solutions in powers of x of the DE. $x^2 y'' - (x^2 + x)y' + y = 0$. (10)

Q. No. 6. (a) Classify general second order partial differential equation (PDE) into elliptic, parabolic and hyperbolic form. Discuss the nature of the PDE $(1 - x^2)u_{xx} - 2xyu_{xy} + (1 - y^2)u_{yy} = 0$ at each $(x, y) \in R^2$. (10)

(b) Use the method of separation of variables to find the solution $u(x, t): [0, T] \times [0, L] \rightarrow R$ to the initial/boundary value problem
 $u_t(x, t) = u_{xx}(x, t)$ for $0 < t \leq T$ and $0 \leq x \leq L$,
 $u(x, 0) = f(x)$, for $0 \leq x \leq L$,
 $u(0, t) = u(L, t) = 0$, for $0 < t \leq T$,
where $f: [0, L] \rightarrow R$ is a known function. (10)

Q. No. 7. (a) Use Simpson's 3/8 rule to estimate the integral $\int_1^3 (x^3 - 2x^2 + 7x - 5)dx$. (10)

By comparing your answer with exact value, find the error.

(b) Solve the system of equations by Jacobi iterative method. (10)
 $10x + 3y + z = 19, \quad 3x + 10y + 2z = 29, \quad x + 2y + 10z = 35$

Q. No. 8. (a) In the following table values of $y = x + \sin x^2$ are tabulated (10)

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$f(x)$	1.84147	2.03562	2.19146	2.29290	2.32521	2.27807	2.14935

Construct a difference table and estimate $f(1.04)$ and $f(1.57)$.

(b) Use trapezoidal and Simpson's 1/3 rules to approximate $\int_0^{\pi/2} \sin^2(x)dx$. Find a maximum bound for the error in each case. Compare your approximations with the actual result. (10)



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Q. No. 1. (a) Let $u=[y, z, x]$ and $v=[yz, zx, xy]$, $f = xyz$ and $g = x + y + z$. Find $\text{div}(\text{grad}(fg))$. (10)

(b) Evaluate $\int_C F(r) \cdot dr$ counter clockwise around the boundary C of the region R by Green's theorem, where
 $F = [y, -x]$, C the circle $x^2 + y^2 = 1/4$ (10)

Q. No. 2. (a) Three forces P, Q, R , acting at a point, are in equilibrium, and the angle between P and Q is double of the angle between P and R . Prove that $R^2 = Q(Q - P)$. (10)

(b) Find the centre of mass of a semi-circular lamina of radius a whose density varies as the square of the distance from the centre. (10)

Q. No. 3. (a) A particle moves in such a way that its position vector at time t is (10)
 $r = (a \cos nt)\mathbf{i} + (b \sin nt)\mathbf{j}$,

Where a, b, n are constants and $a > b > 0$. Show that the path of the particle is an ellipse of semi-major and minor axes a, b respectively, and that the field of force is directed towards the centre of the ellipse. Also find the maximum speed.

(b) An aeroplane is flying with uniform speed v_0 in an arc of a vertical circle of radius a , whose centre is at a height h vertically above a point O of the ground. If a bomb is dropped from the aeroplane when at a height Y and strikes the ground at O , show that Y satisfies the equation (10)
 $KY^2 + Y(a^2 - 2hK) + K(h^2 - a^2) = 0$,

Where $K = h + \frac{ga^2}{2v_0^2}$.

Q. No. 4. (a) Solve the given initial-value problem. Give the largest interval I over which the solution is defined. (10)

$$xy' + y = e^x, y(1) = 2.$$

(b) Find the general solution of the given higher-order differential equation. (10)
 $y''' - 4y'' - 5y' = 0$

Q. No. 5. (a) Find two power series solutions of the given differential equation about the ordinary point $x=0$. (10)

$$y'' - 2xy' + y = 0.$$

(b) Find the general solution of the given Bessel's equation on $(0, \infty)$. (10)
 $x^2y'' + xy' + (9x^2 - 4)y = 0$

APPLIED MATHEMATICS

- Q. No. 6. (a)** Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π . Show the details of your work. **(10)**

$$f(x) = \begin{cases} x, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$$

- (b)** Find $u(x,t)$ for the string of length $L=1$ and $c^2=1$ when the initial velocity is zero and the initial deflection with small k (say, 0.01) is $kx(1-x)$. **(10)**

- Q. No. 7. (a)** Use the Bisection method to determine an approximation to the root of the given function in the interval $[1,2]$ that is accurate to at least within 10^{-4} . **(10)**

$$f(x) = x^3 + 4x^2 - 10 = 0.$$

- (b)** Values for $f(x) = xe^x$ are given in the following table. Use all the applicable three-point and five-point formulas to approximate $f'(2.0)$. **(10)**

x	1.8	1.9	2.0	2.1	2.2
f(x)	10.889365	12.703199	14.778112	17.148957	19.855503

- Q. No. 8. (a)** Use the Modified Euler method to approximate the solution to each of the following initial-value problem, **(10)**

$$y' = -5y + 5t^2 + 2t, \quad 0 \leq t \leq 1, \quad y(0) = \frac{1}{3}, \text{ with } h = 0.1$$

- (b)** Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on $[1, 2]$. Use $p_0 = 1$. **(10)**



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COMPETITIVE EXAMINATION-2023 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE:** (i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

- Q. No. 1.** (a) Forces of magnitudes P , $2P$, $3P$, $4P$ act respectively along the sides AB , BC , CD , DA of a square $ABCD$, of side a and forces each of magnitude $(8\sqrt{2})P$ act along the diagonals BD , AC . Find the magnitude of the resultant force and the distance of its line of action from A . (10)
- (b) A uniform rod AB of length a and weight W is freely hinged to a vertical wall at A and is maintained in equilibrium by a light string of length a fastened to B and to a point C at a distance b vertically above A . Prove that the reaction at the hinge A is (10)

$$W \frac{\sqrt{a^2 + 2b^2}}{2b}$$

and find the tension in the string.

- Q. No. 2.** (a) Use Runge-Kutta method of order two to solve the following differential equation at $x=1.2$ by taking $h=0.1$ (10)
- $$\frac{dy}{dx} = \frac{3x+y}{x+2y} \quad y(1) = 1.$$
- (b) Find the first and second derivatives of $f(x)$ at $x = 3$ from the following data using Newton's forward difference interpolation formula (10)

x	3	3.5	4	4.5	5	5.5
$f(x)$	4.1023	5.1047	8.1971	9.1096	4.1122	6.1148

- Q. No. 3.** (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (8)
- (b) Show that (6)
- $$\nabla r^n = nr^{n-2}r$$
- (c) Find the total work done in a moving particle in a force field given by (6)
- $$F = 3xy \mathbf{i} - 5z \mathbf{j} + 10x \mathbf{k}$$
- along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

- Q. No.4.** (a) A particle P moves in a plane in such a way that at any time t , its distance from a fixed point O is $r = at + bt^2$ and the line connecting O and P makes an angle $\theta = ct^{\frac{3}{2}}$ with a fixed line OA . Find the radial and transverse components of the velocity and acceleration of the particle at $t = 1$. (10)
- (b) Solve the following Bernoulli's equation (10)
- $$x \frac{dy}{dx} + y = \frac{1}{y^2}$$

- Q. No. 5.** (a) Solve the following differential equation (10)
- $$x dy = (x \sin x - y) dx$$
- (b) Find the general solution of the higher order differential equation (10)
- $$y''' + 8y'' = -6x^2 + 9x + 2$$

APPLIED MATHEMATICS

Q. No. 6. (a) Find solution of $4y'' + y = 0$ in the form of power series in x . **(10)**

(b) Solve the following differential equation by variation of parameters **(10)**

$$y'' - 4y' + 4y = (x + 1)e^{2x}$$

Q. No. 7. (a) Find real root of the equation $2x - 3 \sin(x) - 5 = 0$ up to 4 decimal places by secant method. **(10)**

(b) Solve the following system of equations by Gauss Seidel method. Perform only five iterations. **(10)**

$$8x_1 - x_2 - x_3 = 6$$

$$x_1 + 6x_2 + x_3 = 8$$

$$x_1 - x_2 + 5x_3 = 5$$

Q. No. 8. (a) Expand $f(x) = \sin x$, $0 < x < \pi$, in a Fourier cosine series. **(10)**

(b) Use the method of separation of variables to find the solution of the following boundary value problem **(10)**

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \leq x \leq a, \quad 0 \leq y \leq b$$

$$u_x(0, y) = 0, \quad u_x(a, y) = 0,$$

$$u_y(x, b) = 0, \quad u(x, 0) = f(x).$$



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2024 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT
APPLIED MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS

MAXIMUM MARKS = 100

- NOTE: (i)** Attempt only **FIVE** questions in all. **ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

Q. No. 1 (a) Expand a fourier series of $f(x) = x^2, 1 < x < 2$ (10)

(b) Find equation of integral surface of the differential equation $2y(z-3)p + (2x-z)(10)$
which passes through the circle $x^2 + y^2 = 2x, z = 0$.

Q. No. 2 (a) Solve the higher order differential equation $y'''' + y'' = 3x^2 + 4\sin x - 2\cos x$ (10)

(b) Solve the initial value problem $y'' - 8y' + 15y = 9xe^{2x}, y(0) = 5, y'(0) = 10$ (10)

Q. No. 3 (a) Solve the equation $4u_{xx} + 5u_{xy} + u_{yy} + u_x + u_y = 2$, also find its canonical form. (10)

(b) Prove that $\int_{-1}^1 x^n P_n(x) dx = \frac{2^{n+1} (n!)^2}{(2n+1)!}$, where $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n$ is Legendre polynomial of degree n . (10)

Q. No. 4 (a) Verify the divergence theorem for $A = 4xi - 2y^2j + z^2k$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$. (10)

(b) State and prove Stoke's theorem. (10)

Q. No. 5 (a) Using the modified Euler's method, obtain the solution of the differential equation (10)

$$\frac{dy}{dt} = t + \sqrt{y} = f(t, y)$$

with initial condition $y_0 = 1$ at $t_0 = 0$ for the range $0 \leq t \leq 0.6$ in step of 0.2 .

(b) Find the real roots of equation $4x + \cos x + 2 = 0$ by using Newton Raphson method, correct to four decimal places. (10)

Q. No. 6 (a) Solve the system of linear equations by Guass-Seidel iterative method and perform the first three iterations of (10)

$$\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned}$$

APPLIED MATHEMATICS

- (b) Solve the following Van der Pol's equation $y'' - (0.1)(1 - y^2)y' + y = 0$, (10)

using fourth order Runge-Kutta method for $x = 0.2$, with the initial values

$$y(0) = 1, \quad y'(0) = 0.$$

- Q. No. 7** (a) Find the law of force for a particle moving in an orbit, $r = \frac{l}{1 - e \cos \theta}$, where l is (10)

semi latus rectum and e is eccentricity.

- (b) Prove that the speed required to project a particle from a height h to fall a (10)

horizontal distance a from the point of projection is at least $\sqrt{g(\sqrt{a^2 + h^2} - h)}$.

- Q. No. 8** (a) Find the radial and transvers components of velocity moving along a curve $ax^2 + b$ (10)

at any time t if the polar angle $\theta = ct^2$.

- (b) Find the centroid of the surface formed by the revolution of the cardioide (10)

$r = a(1 + \cos \theta)$ about the initial line.



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TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT
APPLIED MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100
NOTE: (i) Attempt only FIVE questions in all. ALL questions carry EQUAL marks. (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places. (iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper. (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed. (v) Extra attempt of any question or any part of the attempted question will not be considered. (vi) Use of Calculator is allowed.	

Q. No. 1 (a) (i) Prove that $\nabla r^n = nr^{n-2} \vec{r}$, where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$. (10)

(ii) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$, then prove that $\vec{a}$ and $\vec{c}$ are parallel.

(b) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$. (10)

Q. No. 2 (a) Find the tangential and normal components of acceleration of a point describing the ellipse (10)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ with uniform speed } \vec{V}, \text{ when the particle is at } (0, b).$$

(b) Find the solution of initial value problem by separation of variables (10)

$$\sqrt{1-y^2} dx - \sqrt{1-x^2} dy = 0, \quad y(0) = \frac{\sqrt{3}}{2}$$

Q. No. 3 (a) Find the general solution of the given differential equation by variation of parameters. (10)

$$3y'' - 6y' + 6y = e^x \sec x$$

(b) Find the power series solution of $(x^2 + 1)y'' + xy' - y = 0$ (10)

Q. No. 4 (a) Forces $2\vec{BC}$, $\vec{CA}$, $\vec{BA}$ act along the sides of a triangle ABC . Show that their resultant is $6\vec{DE}$. Where D bisects BC and E is a point on CE such that $CE = \frac{1}{3}CA$. (10)

(b) Find the center of mass of the surface generated by the revolution of the arc of the parabola, lying between the vertex and the latus rectum, about the x -axis. (10)

Q. No. 5 (a) Obtain the Fourier series over the indicated interval for the given function. (10)

$$f(x) = 3\pi + 2x, \quad -\pi < x < 0, \\ = \pi + 2x, \quad 0 < x < \pi$$

(b) Solve the boundary value problem (10)

$$u_{xx} + u_{yy} = 0, \quad 0 < x < a, \quad 0 < y < b, \\ u(0, y) = 0, \quad u(a, y) = 0, \quad 0 \leq y \leq b \\ u(x, 0) = 0, \quad u(x, b) = f(x), \quad 0 \leq x \leq a.$$

Q. No. 6 (a) Use Newton's Raphson method to find the solution accurate to within 10^{-4} (corrected upto four decimal places) for the given problem. (10)

$$x - \cos x = 0, \quad [0, \pi/2].$$

(b) Solve the system of linear equations using Gauss Seidel method (with three digit rounding arithmetic) (10)

$$3x_1 + 4x_2 - x_3 = 8 \\ 5x_1 + 3x_2 + 2x_3 = 17 \\ -x_1 + x_2 - 3x_3 = -8$$

APPLIED MATHEMATICS

- Q. No. 7** (a) Use Euler's method to approximate the solution of the initial value problem. (10)
 $y' = 1 + y/x, \quad 1 \leq x \leq 2, \quad y(1) = 2, \quad \text{with } h = 0.25$
- (b) Using Green's theorem, evaluate $\int_C \vec{F}(\vec{r}) \cdot d\vec{r}$ counter clock wise around the (10)
boundary curve C of the region R , where $\vec{F} = \left[\frac{1}{2}xy^4, \frac{1}{2}x^4y \right]$, R the rectangle
with vertices $(0, 0), (3, 0), (3, 2), (0, 2)$.
- Q. No. 8** (a) Evaluate the Integral $\int_1^3 \frac{1}{x^2} dx$, Using Trapezoidal Rule for five points (corrected (10)
upto two decimal places).
- (b) Find the D'Alembert solution of the wave equation $u_{xx} = \frac{1}{c^2} u_{tt}$ subject to the (10)
Cauchy Initial conditions $u(x, 0) = f(x), \quad u_t(x, 0) = g(x)$.



FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2026 FOR RECRUITMENT
TO POSTS IN BS-17 UNDER THE FEDERAL GOVERNMENT
APPLIED MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100
NOTE: (i) Attempt only FIVE questions in all. ALL questions carry EQUAL marks. (ii) All the parts (if any) of each Question must be attempted at one place instead of at different places. (iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper. (iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed. (v) Extra attempt of any question or any part of the attempted question will not be considered. (vi) Use of Calculator is allowed.	

Q. No. 1 (a) Prove $\nabla f(r) = \frac{f'(r)}{r} \vec{r}$, and evaluate U for $\nabla U = 2r^4 \vec{r}$. (8)

(b) If $\vec{A}$ and $\vec{B}$ are differentiable vector function of position (x, y, z) , find $\nabla \cdot (\vec{A} \times \vec{B})$. (7)

(c) If $\vec{A} = x^2 y z i - 2 x z^3 j + x z^2 k$ and $\vec{B} = 2 z i + y j - x^2 k$, find $\frac{\partial^2}{\partial x \partial y} (\vec{A} \times \vec{B})$. (5)

Q. No. 2 (a) What is the maximum range possible for a projectile fired from a cannon having muzzle velocity 1 mile/sec? What is the height reached in this case? (10)

(b) Find the radial and transverse components of the velocity of the particle moving along the curve $ax^2 + by^2 = 1$ at any time t , if the polar angle $\theta = ct^2$. (10)

Q. No. 3 (a) Verify that the linear differential equation (5)

$$(1 + x^2) \frac{dy}{dx} + 2xy = e^{x^2}$$

can be solved by setting $y = ve^{x^2}$. Find the equation satisfied by v .

(b) Explain how a first-order linear differential equation can be used to model pollution accumulation in a lake with constant inflow and outflow rates. (5)

(c) Find the solution of the following differential equation (10)

$$y''' + 8y = 2x - 5 + 8e^{-2x}; \text{ with the conditions}$$

$$y(0) = -5, \quad y'(0) = 3, \quad y''(0) = -4$$

Q. No. 4 (a) Find the general solution of the following differential equation on interval $(0, \infty)$ (10)

$$16x^2 y'' + 16xy' + (16x^2 - 1)y = 0$$

(b) Solve the following PDEs using transformation (10)

$$u_t + u_x + u = 0; \text{ for } x > 0, t > 0,$$

$$u(0, t) = \sin(t); u(x, 0) = 0.$$

Q. No. 5 (a) Model the one-dimensional wave equation. (8)

(b) A stretched string of length L is lying along x -axis and is fixed at both ends $x = 0$ and $x = L$. Find the deflection $u(x, t)$ of the string at any time t , if initial displacement $u(x, 0) = f(x) = Lx - x^2$ and the initial velocity is $u_t(x, 0) = g(x) = 4$. (12)

Q. No. 6 (a) Find the Fourier series of the function (10)

$$f(x) = \left(1 - \frac{|x|}{a}\right) H\left(1 - \frac{|x|}{a}\right),$$

Where $H(x)$ is the Heaviside unit step function defined by

$$H(x) = \begin{cases} 1, & \text{for } x > 0 \\ 0, & \text{for } x < 0 \end{cases}$$

APPLIED MATHEMATICS (2026)

- (b) Evaluate the integral $I = \int_0^3 \frac{1}{1+x^4} dx$ by using (10)
- (i) Trapezoidal rule
- (ii) Simpson's 1/3 rule by taking $h = 1/4$.

- Q. No. 7** (a) Use the Modified Euler method to solve the IVP: (10)

$$y' = y - x^2 + 1; \quad y(0) = 0.5$$

for $0 \leq x \leq 2$ with step size $h = 0.5$.

Compare with the exact solution.

- (b) Implement appropriate technique to solve the following linear system. (10)

$$3x_1 - x_2 + x_3 - 3 = -1$$

$$-x_1 + 3x_2 - x_3 = 7$$

$$x_1 + x_2 - 3x_3 = -7$$

- Q. No. 8** (a) Set up Newton's scheme of iteration for finding the square root of a positive number N and evaluate $\sqrt{14}$. (10)

- (b) Find the first four iterations of the equation $f(x) = x - 0.8 - 0.2\sin x$ in the interval $\left[0, \frac{\pi}{2}\right]$ using Newton-Raphson Method. (10)
